C ODEBREAKER

Class discussion

QUESTION 1

Strategies for codemaker

What did you do as codemaker?

- When you were playing as codemaker, what strategy did you use to pick a code?

QUESTION 2

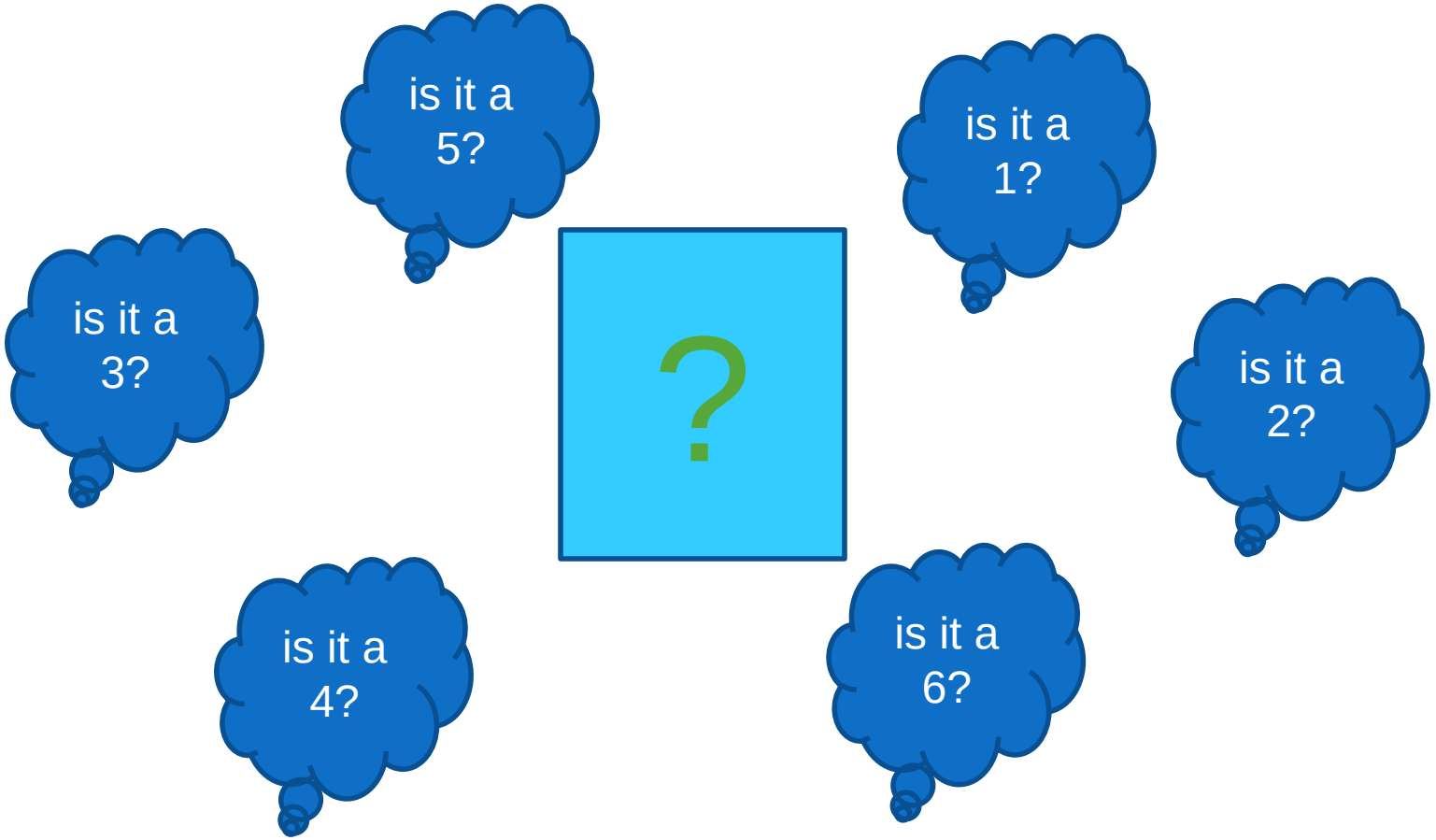
number of codes

How many codes are there?

- Anyone have a guess how many different possible codes there are?

Number of codes

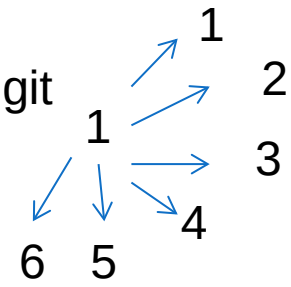
- If we used just 1 digit, there would be 6 possible codes.



What if there are 2 digits?

If 1st digit is 1

6 possible choices for 2nd digit



6 possibilities for first number

6 possibilities for second number

6 * 6 = 36 possible combinations

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

What if more than 2 digits in code?

- 6 possibilities if only 1 digit
- 6 * 6 = 36 (6^2)
- 6 * 6 * 6 = 216 (6^3)
- 6 * 6 * 6 * 6 = 1296 (6^4) – Mastermind

So there are more than 1000 possible codes!

What if valid numbers in code changes?

Assume that instead of 1-6, each number can just be 1-4

- 4 if only one digit
- 4 * 4 = 16 (4^2)
- 4 * 4 * 4 = 64 (4^3)
- 4 * 4 * 4 * 4 = 256 (4^4)

Or if each number could be 1-10

- 10 * 10 * 10 * 10 = 10,000 (10^4)

QUESTION 3

Codebreaker strategy

What's a good strategy for the codebreaker?

- How would you choose your first guess?
- What would you do next?

A possible strategy

Start with a code that has only 2 digits

e.g., 4455, 1122, 2266

Let’s walk through some examples to see why this might be good.

Assume the code is **1231**

Code is 1231 – Guess 1

- First guess is **4455**
- Response is 0-0
- What do we know?
  - **We should never guess a 4 or a 5 again!**

Code is 1231 – Guess 2

- We know we should not guess a 4 or a 5.
- Let's try **1122**

- Response is 1-2

- What do we know?
  - One correct digit in right location, two correct digits in wrong location
  - Total of three correct digits
  - The code must include either two 1s or two 2s
  - The code must also include either a 3 or a 6 OR one more 1 or 2 (e.g., could be a code like 1211 or 2221)

Code is 1231 – Guess 3

- What should we do now?
  - There are various options that are valid
  - We might try other codes to see whether 3 or 6 are included
  - We might try to rearrange numbers to figure out which digit was in the correct location, and move others in the correct location

- Let's try **3333**
- The response is 1-0. So we know the code contains a 3 (in some location)

- NOTE: more advanced players would likely avoid this guess, as it can't possibly win. But it is a simple way to find out for sure it the digit 3 is included. An alternative approach is covered later.

Code is 1231 – Guess 4

- We know the code contains 1, 2 and 3. It does not contain 4 or 5. Do we need to try 6?

- 1122 response was 1-2. So three of the digits are 1 and 2
- 3333 response was 1-0. So one digit must be a 3
- That's four digits…. so no need to try 6

- Let's rearrange our first guess and try **1221**
- The response is 3-0

Code is 1231 – Guess 5

- The response to 1221 was 3-0 so three digits are in the correct location
- We know that the code also contains a 3

- Let’s try putting 3 in one location at a time, first **3221**
- The response is 2-1

- So 3 is NOT the first digit

Code is 1231 – Guess 6

- 1221 response was 3-0
- 3221 response was 2-1

- Let’s try 3 in the next location: **1321**
- The response is 2-2

- So 3 is NOT the second digit

Code is 1231 – Guess 7

- 1221 response was 3-0
- 3221 response was 2-1
- 1321 response was 2-2

- Let’s try 3 in the next location: **1231**
- The response is 4-0

- HURRAY! We found the code in 7 guesses.

Alternative sequence for 1231

Here's a more effective sequence, but it requires slightly more reasoning, especially for guess 3.

1. 4455 => 0-0 (no 4 or 5 in code)
2. 1122 => 1-2 (contains 1,1,2 or 1,2,2 - in some order)
3. 1222 => 2-0 (contains 12_ _)
4. 1213 => 2-2 (we know the digits, fix the order)
5. 1231 => 4-0!

QUESTION 4

Strategies for computer players

Are computers better at this?

- Do you think a computer program could be written that would perform better at this task than we do?

- Try this (optional, copy and paste into browser):
  http://csunplugged.mines.edu/mastermind/mastermind.cgi

A computer strategy

Here's one *algorithm* (step-by-step process) a computer might use.

Remember there are 1296 possible codes!

1. Select a random code and ask the codemaker for response.
2. If the codemaker gives a winning response (i.e., 4-0), we have won. Stop.
3. Discard all elements which would no longer be possible under the codemaker's response.
   - How would we know which ones are no longer possible?
4. Goto step 1

How does the computer eliminate codes?

Assume the secret code is 1231.

After a few guesses, assume the only possible codes that remain are:

1222

1322

1221

1231

Computer randomly selects 1222.

What would the response be?

Eliminating Codes

- Right! The response would be 2-0
- So is 1222 correct? No!
- Which other codes could be correct, given that response?

Eliminating Codes

- Could the answer be 1322?
- Remember the computer guessed 1222.
- If correct code is 1322, then a “guess” of 1222 would get a “response” of 3-0 (3 digits in correct location, 1_22).
- BUT, response was 2-0, not 3-0. So remove 1322!

Eliminating Codes

- Could the answer be 1221?
- Remember the computer guessed 1222.
- If correct code is 1221, then a “guess” of 1222 would get a “response” of 3-0 (3 digits in correct location, 122_).
- BUT, response was 2-0, not 3-0. So remove 1221!

Eliminating Codes

- Could it be 1231?
- Remember the computer guessed 1222.
- If correct code is 1231, then a “guess” of 1222 would get a “response” of 2-0 (2 digits in correct location, 12__).
- Response was 2-0. So 1231 might be the right answer. Keep it!

Eliminating codes

1222 – eliminated (guessed, not correct)

1322 – eliminated (based on response)

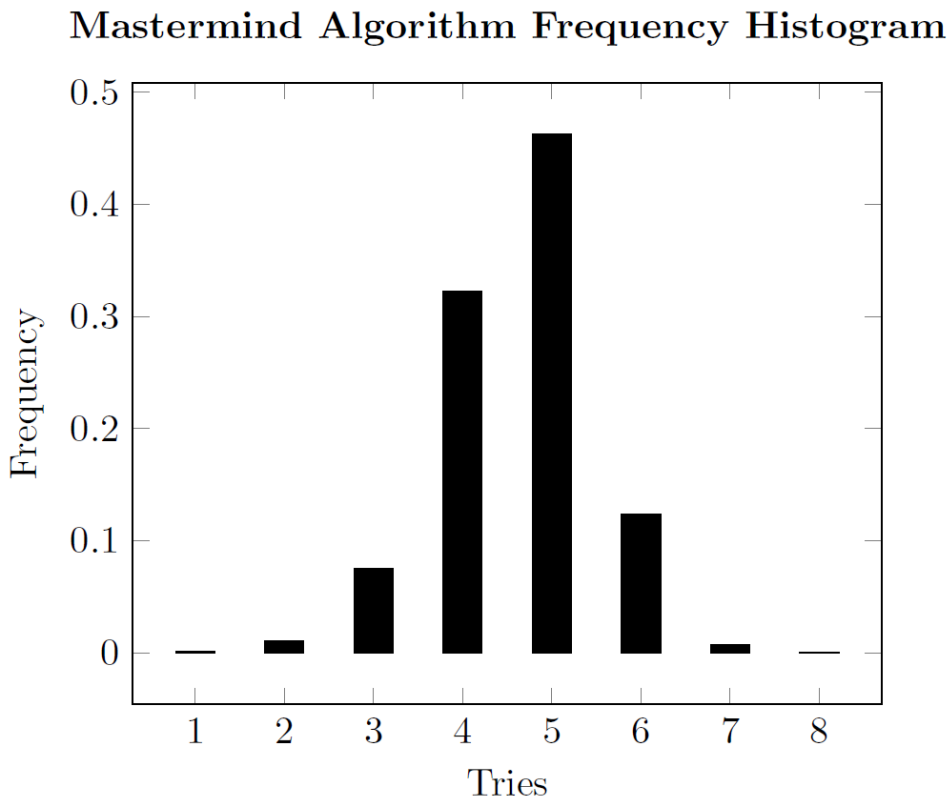
1221 – eliminated (based on response)

1231 – still a possible answer

Only 1 possible answer remaining – so it must be 1231!

How well does this strategy preform?

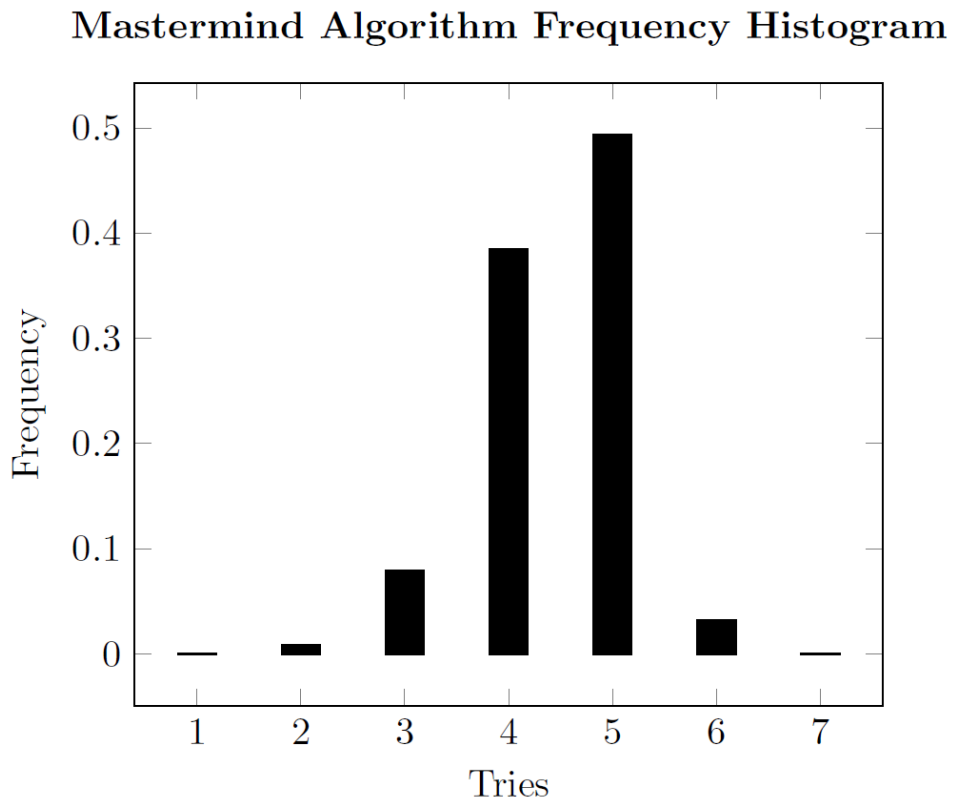
- Using a computer to simulate many Computer vs Computer gameplays, here's how frequent each number of guesses takes:



Guesses in five tries or less ~85% of the time!

Can we do better?

- Making an “educated guess” about which code to choose (rather than just random):



Guesses in five tries or less ~97% of the time!

This is what the computer program demo uses.

Can we do *even better*?

- Minimax strategy (used in economics and AI applications) can win in 5 tries or less 100% of the time.
  - Downside: Very computationally expensive, especially when the game is extended to more digits
  - Downside: You can get a better average number of moves when you don’t impose a hard limit of 5 moves (think about this: why might it be so?)

Good for Computers, not Humans

- The computer can use *exhaustive search* – generate all possible combinations then eliminate them
- Would you want to try to do exhaustive search?
- Computer programs have been developed that are guaranteed to solve the Mastermind problem in at most 5 guesses (and can sometimes break the code in fewer guesses).