Stationary Distributions for the Random Waypoint Mobility Model[*†‡]

William Navidi and Tracy Camp
Department of Mathematical and Computer Sciences
Colorado School of Mines
Golden, Colorado 80401
wnavidi@mines.edu; tcamp@mines.edu

April 12th, 2003

Abstract

In simulations of mobile ad hoc networks, the probability distribution governing the movement of the nodes typically varies over time, and converges to a "steady-state" distribution, known in the probability literature as the *stationary distribution*. Some published simulation results ignore this initialization discrepancy. For those results that attempt to account for this discrepancy, the practice is to discard an initial sequence of observations from a simulation in the hope that the remaining values will closely represent the stationary distribution. This approach is inefficient and not always reliable. However, if the initial locations and speeds of the nodes are chosen from the stationary distribution, convergence is immediate and no data need be discarded. We derive the stationary distributions for location, speed, and pause time for the random waypoint mobility model. We then show how to implement the random waypoint mobility model in order to construct more efficient and reliable simulations for mobile ad hoc networks. Simulation results, which verify the correctness of our method, are included. In addition, implementation of our method for the NS-2 simulator is available.

Keywords: simulation of mobile ad hoc networks, random waypoint mobility model, mobility models

[*]This work was partially supported by National Science Foundation grant ANI-0208352.
[†]Research group's URL is *http://toilers.mines.edu*
[‡]Reference for this manuscript: Technical Report MCS-03-04, The Colorado School of Mines, April 2003.

1 Introduction

Mobile ad hoc networks are often studied through simulation, and their performance can depend heavily on the mobility model that governs the movement of the nodes [1]. In most cases, the probability distribution of the initial locations and speeds of the nodes differs from the distribution at later points in the simulation. In fact, it is generally true that the probability distributions of both location and speed vary continuously over time, and converge to a "steady-state" distribution, known in the probability literature as the *stationary* distribution. At any given point in the simulation, the distribution of location and speed is a weighted average of the initial distribution and the stationary distribution, with the weight shifting from the initial distribution to the stationary distribution as the simulation progresses.

Because the distributions of location and speed vary as a simulation progresses, the performance of the network can vary as well. In particular, network performance early in a simulation may differ substantially from the performance later in the simulation [2]. Up to now, the primary method for dealing with this problem (when the problem is addressed at all) has been to discard an initial sequence of observations [3]. The hope is that the values observed for location and speed past this initial sequence will have been sampled approximately from the stationary distribution. This approach has two drawbacks. First, it is inefficient, since it requires the discarding of data. Second, and more importantly, it is difficult to know just how long a sequence one needs to discard in order to be safely near stationarity. As we will show below, if the minimum speed is low, convergence can take more than 1000 seconds of simulation time.

We focus our discussion on the random waypoint mobility model [4, 5]. In this model, each node is assigned an initial location (x_0, y_0), a destination (x_1, y_1), and a speed S. The points (x_0, y_0) and (x_1, y_1) are chosen independently and uniformly on the region in which the nodes move. The speed is chosen uniformly on an interval (v_0, v_1), independently of both the initial location and destination[1]. After reaching the destination, a new destination is chosen from the uniform distribution, and a new speed is chosen uniformly on (v_0, v_1), independently of all previous destinations and speeds. Nodes may pause upon reaching each destination, or they may immediately begin traveling to the next destination without pausing. If they pause, the pause times are chosen independently of speed and location.

Virtually all published simulation results that use the random waypoint mobility model begin with the nodes placed uniformly in the simulation area. Of course, this initial random distribution of nodes is *not representative of the manner in which nodes distribute themselves when moving*. The stationary distributions of location and speed in the random waypoint mobility model are in fact quite different from the uniform distribution. In particular it has been noticed [2, 7, 8] that the stationary distribution of the location of a node is more concentrated near the center of the region in which the nodes move, since nodes traveling between uniformly chosen points spend more time near the center than near the edges. Yoon et al. [2] noticed that the stationary distribution of the speed differs from the uniform as well, and showed in particular that if the minimum speed v_0 is taken to be 0, the mean node speed approaches 0. A variant of the random waypoint mobility model is presented in [11], but the stationary distribution for this model differs from the uniform as well. Blough et al. [13] determined that if the nodes move according to a random walk rather than according to the random waypoint mobility model, the stationary distribution of the location is close to uniform.

One implementation of the random waypoint mobility model (*setdest*) begins with a pause at the initial location [5, 6]. Another implementation (*mobgen*) begins with the nodes moving [3]; thus, the first pause occurs upon reaching the first destination. Once the initial locations are uniformly chosen, simulations that use *setdest* (or a variant of it) have nodes begin paused at their initial loca-

[1]In practice, the distribution for speed is usually uniform. In principle, any distribution is possible. We discuss alternatives for speed in Section 5.

tions (the pause time is chosen from a uniform distribution). In other words, each node remains in its initial uniformly distributed position for a given initial pause time. Simulations that use *mobgen* (or a variant of it) begin with nodes moving from their initial locations to their first destinations immediately. For this reason we suspect that simulations using *setdest* may take longer to converge than simulations using *mobgen*, although we have not verified this.

In [1], the authors present three approaches to the initialization problem. The first is to save the locations of the nodes after a simulation has executed long enough to be past the period of high variability, and use this position file as the initial starting point of the nodes. By creating many such position files, and starting each simulation trial with a different one, each simulation trial is started from a distribution close to stationarity. The second approach, which is essentially equivalent to the first, is to discard an initial number of seconds of simulation time produced by the random waypoint mobility model in each simulation trial. The authors suggest that discarding 1000 seconds of simulation time (regardless of the node's speed) will ensure that the initialization problem is removed. While this is true for many simulations, we show below that convergence can take more than 1000 seconds of simulation time if the minimum speed is low. This points out one of the difficulties with these two approaches; it is difficult to know just how long a sequence one needs to discard. The third approach proposed in [1] is to assign initial positions and speeds to the nodes according to the distribution they will come to have over time, i.e., the stationary distribution. In this paper, we will describe the stationary distribution for the random waypoint mobility model, and show how to implement this approach in simulations.

We derive the stationary distributions for speed, location, and pause time for a node moving in a rectangular area under the random waypoint mobility model. If the initial speed, location (and pause time, if applicable) are sampled from the stationary distribution rather than the uniform distribution, convergence to stationarity is immediate, and no data need be discarded. Note that only the initial location and speed (and pause time, if applicable) need to be sampled from the stationary distribution; all subsequent node destinations, speeds and pause times should be sampled from the uniform distribution.

For the sake of simplicity, we assume that the network region is the unit square. It is straightforward to adjust the scaling to apply our results to any rectangular network. For example, if the pause time is zero, the location of the node in the unit square is (x, y), and the simulation area is a rectangle of size 300 m $\times$ 600 m, then the node's location in the simulation area is $(300x, 600y)$. For a non-rectangular region, the stationary distribution of location will differ from that of a rectangular network, and must be derived separately.

In Section 2 we derive the stationary distribution and give implementation details of the random waypoint mobility model when the pause time is 0. In Section 3, we include pause time in the derivation of the distribution and the implementation discussion. Simulation results, presented in Section 4, illustrate that our proposed implementation for the random waypoint mobility model without pausing (unlike the *setdest* and *mobgen* implementations) simulates the stationary distribution throughout the simulation time period. Results for the random waypoint mobility model with pausing are similar and, therefore, not presented. In Section 5 we present a generalization of our method that can be used to produce any desired distribution (not necessarily uniform) for speed. Finally, in Section 6 we state our conclusions.

2 Without Pausing

2.1 Stationary Distribution (without Pausing)

We derive the stationary distributions for both speed and location when the pause time is zero. We begin with speed. At any time t, the node has a speed, which we denote by S. For any positive number M, define the function $F_M(s)$ to be the proportion of time in the interval $(0, M)$

that the speed of the node is less than or equal to s. Define $F(s) = \lim_{M\to\infty} F_M(s)$. Then $F(s)$ is the cumulative distribution function of the stationary distribution of S. We will compute the probability density function, $f(s)$, for the stationary distribution.

Since S is chosen independently of location, it is independent of path length. Therefore, without loss of generality, we may assume the path length is 1 when computing $f(s)$. If the node is traveling at speed s, the time spent on a path of length 1 is $1/s$. Therefore $f(s)$ is proportional to $1/s$. Since $\int_{v_0}^{v_1} f(s)\,ds = 1$,

$$f(s) \quad = \quad \begin{cases} \dfrac{1}{s\log(v_1/v_0)} & v_0 < s < v_1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Note that Equation (1) is valid only when the minimum speed v_0 is greater than 0.

To find the probability density function for location, note that at any time t, the node is traveling on a straight line path between two points. Since the speed is constant along the path, the position of the node is uniformly chosen from among the points on the path. Therefore, conditional on the endpoints of the path being (x_1, y_1), and (x_2, y_2), the probability density of the x-coordinate of the node's location is

$$g(x|x_1,x_2) \quad = \quad \begin{cases} \dfrac{1}{|x_2 - x_1|} & x \text{ between } x_1 \text{ and } x_2 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

We express the unconditional probability density function of the x-coordinate as an integral involving the joint density of x_1, x_2, y_1, and y_2. The key is to note that the proportion of time spent by a node on a path of a given length is proportional to the length of the path. Therefore the joint density of the path endpoints (x_1, y_1, x_2, y_2) is given by

$$h(x_1,x_2,y_1,y_2) = k[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} \tag{3}$$

where k is chosen so that the density integrates to 1. The value of k is computed by using the equation

$$1/k = \int_0^1 \int_0^1 \int_0^1 \int_0^1 [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} \, dx_1 \, dx_2 \, dy_1 \, dy_2 \tag{4}$$

(Recall that we assume the simulation region is the unit square.) Numerical integration yields $k = 1.9179$.

The unconditional density of the x-coordinate is given by

$$\begin{aligned} g(x) &= \int_0^x \int_x^1 \int_0^1 \int_0^1 g(x|x_1,x_2) h(x_1,x_2,y_1,y_2) \, dy_1 \, dy_2 \, dx_1 \, dx_2 \\ &+ \int_0^x \int_x^1 \int_0^1 \int_0^1 g(x|x_1,x_2) h(x_1,x_2,y_1,y_2) \, dy_1 \, dy_2 \, dx_2 \, dx_1 \\ &= 2 \int_0^x \int_x^1 \int_0^1 \int_0^1 \frac{k[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}}{x_2 - x_1} \, dy_1 \, dy_2 \, dx_1 \, dx_2 \end{aligned} \tag{5}$$

Since the x-coordinate and y-coordinate are identically distributed, the probability density of the y-coordinate is also given by $g(x)$. It does not seem feasible to compute the integral given in Equation (5) in closed form, although it can be computed numerically for any given value of x. As pointed out by Bettstetter and Wagner [8], there is a slight dependence between the x and y coordinate, so $h(x,y)$ is not given by the product $g(x)g(y)$.

In [9], a close approximation to the joint density $h(x,y)$ of the x and y coordinates is given, and it is suggested that initial locations be sampled from this approximation. In fact, it is not necessary to compute the joint density $h(x,y)$ in order to sample from it; we describe such a sampling method in Subsection 2.2.

2.2 Implementation (without Pausing)

As discussed in Section 1, the primary method for dealing with the initialization problem of the random waypoint mobility model (if the problem is addressed at all) has been to discard an initial sequence of observations. To avoid this inefficiency in discarding data, it is only necessary to sample the initial speed and location from their stationary distributions. Subsequent speeds and locations should then be sampled from the uniform distribution. To show how to sample the initial speed S from the stationary distribution, we compute the cumulative distribution $F(s)$. Let $f(s)$ be as in Equation (1). Then

$$
\begin{aligned}
F(s) &= \int_{v_0}^{s} f(t)\,dt \\
&= \frac{\log(s) - \log(v_0)}{\log(v_1) - \log(v_0)}
\end{aligned}
\tag{6}
$$

The inverse of $F(s)$ is

$$
F^{-1}(u) = \frac{v_1^u}{v_0^{u-1}} \tag{7}
$$

To choose the initial speed S, choose U uniformly on $(0,1)$, then let $S = F^{-1}(U)$.

An initial location can be chosen in two simple steps: choose an initial path and then choose a point on that path uniformly. The probability density of any chosen path is proportional to the expected time spent on that path. Since the speed is independent of the path, the expected length of time spent on any given path is equal to the length of the path divided by the expected speed. Thus, the probability density of any path is proportional to its length.

The initial path is therefore chosen by choosing endpoints (x_1, y_1), and (x_2, y_2) in such a way that the joint probability density of these two points is proportional to the distance between them. A convenient way to do this is by rejection sampling. First choose two points (x_1, y_1) and (x_2, y_2) uniformly on the unit square. Compute the length of the path between these two points, and divide by the length of the longest possible path, which is $\sqrt{2}$. Call this quotient r. Generate a random variable U on $(0,1)$. If $U < r$, then accept (x_1, y_1) and (x_2, y_2) as the endpoints of the initial path. Otherwise, start over again with new values of (x_1, y_1), and (x_2, y_2). Once the endpoints of the path have been determined, the initial location of the node is chosen at random uniformly from the points on the path.

The following seven steps give a step-by-step summary of our procedure:

1. Generate (x_1, y_1), and (x_2, y_2) uniformly on the unit square.

2. Compute $r = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}/\sqrt{2}$.

3. Generate a random value U_1 uniformly on $(0,\ 1)$.

4. If $U_1 < r$, then accept (x_1, y_1), and (x_2, y_2). Otherwise, go to step 1.

5. Generate a random value U_2 uniformly on $(0,\ 1)$.

6. The initial location for the node is $(U_2 x_1 + (1 - U_2) x_2,\ U_2 y_1 + (1 - U_2) y_2)$.

7. The node then travels to (x_2, y_2) at the initially chosen speed. Upon reaching $(x_2,\ y_2)$, subsequent speeds and destinations are chosen from the uniform distribution.

In our simulations, we rejected about 60% of the candidates for the initial path. In other words, we had to generate an average of 2.5 candidate paths for each node to get an acceptable initial path. Since rejection sampling is employed only for the initial path, and not for any subsequent path, the extra time added to the simulation was quite small.

3 With Pausing

3.1 Stationary Distribution (with Pausing)

Suppose S is the speed of a node and (X, Y) is the x and y coordinates of the node. Let $f(s)$ be the stationary density of S, and let $g(x)$ be the stationary density of X (and of Y, since the x-coordinate and y-coordinate are identically distributed) if there is no pausing. The expressions for $f(s)$ and $g(x)$ are given in Equation (1) and Equation (5) respectively. Assume that at each destination a pause time P is chosen according to a probability density function $h(p)$. In practice, $h(p)$ is usually a uniform distribution, but in principle this need not be so. Further assume that P is independent of S, X, and Y.

We begin by computing the proportion of time that the node is paused. We refer to the travel between two consecutive destinations as an *excursion*. Let T be the time spent traveling on an excursion, excluding pause time. By definition of the random waypoint mobility model, the node's movement consists of periods of travel alternating with periods of pausing. Let $E(P)$ denote the expected length of a pause, and let $E(T)$ denote the expected time elapsed in traveling between two pauses. The long-run proportion of time spent paused is

$$P_{pause} = \frac{E(P)}{E(P)+E(T)} \tag{8}$$

The expected pause time, $E(P)$, depends on the distribution from which the pause time is sampled, and is given by

$$E(P) = \int_0^\infty p\,h(p)\,dp \tag{9}$$

To compute $E(T)$, let L be the length of an excursion, and let S be the speed of the node on that excursion. Note that, according to the random waypoint mobility model, S is chosen from a uniform distribution on $(v_0,\ v_1)$ at the beginning of each excursion. Then $T = L/S$ and

$$\begin{aligned} E(T) &= E(L/S) & (10)\\ &= E(L)\,E(1/S) & (11) \end{aligned}$$

since L and S are independent.

We first compute $E(1/S)$:

$$\begin{aligned} E(1/S) &= \int_{v_0}^{v_1} (1/s)\frac{1}{v_1 - v_0}\,ds \\ &= \frac{\log(v_1/v_0)}{v_1 - v_0} & (12) \end{aligned}$$

To compute $E(L)$, note that L is the distance between two points on the unit square chosen independently and uniformly. Therefore

$$E(L) = \int_0^1 \int_0^1 \int_0^1 \int_0^1 [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2} \, dx_1 \, dx_2 \, dy_1 \, dy_2 \tag{13}$$

We compute this integral numerically to obtain $E(L) = 0.521405$. Therefore

$$E(T) = 0.521405 \frac{\log(v_1/v_0)}{v_1 - v_0} \tag{14}$$

The result in Equation (14) has also been shown in [12]. Note that for a square of side a, $E(L) = 0.521405a$. For a rectangular region with sides a and b, $E(L)$ is computed by integrating Equation (13) numerically with limits from 0 to a for x_1 and x_2 and from 0 to b for y_1 and y_2, and dividing the result by a^2b^2.

Now let $f_1(s)$ be the density of S. The conditional density $f_1(s|\text{Not paused})$ is equal to the density $f(s)$ in Equation (1). Furthermore, if the node is paused, the speed is zero with probability 1. Therefore

$$\begin{aligned} f_1(s) &= f_1(s|\text{Paused})P_{pause} + f_1(s|\text{Not paused})(1 - P_{pause}) \\ &= \begin{cases} 0 & s < 0 \\ P_{pause} & s = 0 \\ 0 & 0 < s < v_0 \\ \dfrac{1 - P_{pause}}{s \log(v_1/v_0)} & v_0 \leq s \leq v_1 \\ 0 & s > v_1 \end{cases} \end{aligned} \tag{15}$$

Let $g_1(x)$ be the density of X (and of Y, since the x-coordinate and y-coordinate are identically distributed). Then

$$g_1(x) = g_1(x|\text{Paused})P_{pause} + g_1(x|\text{Not paused})(1 - P_{pause}) \tag{16}$$

where $g_1(x|\text{Not paused})$ is equal to the density $g(x)$ in Equation (5), and $g_1(x|\text{Paused})$ is the uniform density on $(0,\ 1)$, since the coordinates of path endpoints are uniformly distributed.

3.2 Implementation (with Pausing)

As discussed in Section 1, simulations that use *setdest* have nodes begin in a paused state and simulations that use *mobgen* have nodes begin in a moving state. However, to simulate the stationary distribution of the random waypoint mobility model with non-zero pause time, some nodes should begin in a paused state and other nodes should begin in a moving state. Therefore, to begin a simulation at the stationary distribution with pausing, the first thing to do is to determine, for each node, whether the node will begin in a paused state or in a moving state.

To accomplish this goal, choose U uniformly on (0,1) for each node. If $U < P_{pause}$ the node will begin from a paused state; otherwise it begins in motion. If the node begins in motion, the procedure for choosing the initial position and speed are the seven steps given in Section 2.2 for the implementation without pausing.

If the node is to begin from a paused state, it is necessary to determine the length of time, P_0, of its initial pause. To achieve stationarity, P_0 must be the length of time from a point chosen at random from within a pause period until the end of that pause period. Recall that $h(p)$ denotes the probability density function of the pause time P. Let $H(p)$ be the cumulative distribution function associated with $h(p)$. By a fundamental result in renewal theory [10], the cumulative distribution function of P_0 is

$$H_0(p) = \frac{\int_0^p [1 - H(t)]\,dt}{E(P)} \tag{17}$$

To sample P_0 from the cumulative distribution function $H_0(p)$, it is necessary to compute the inverse H_0^{-1}. Then choose U uniformly on (0,1) and let $P_0 = H_0^{-1}(U)$. The initial position (x_1, y_1) is chosen uniformly on the unit square. The node remains at $(x_1,\ y_1)$ for a length of time equal to

P_0. The initial speed of the node, once the period P_0 is over, is chosen uniformly on (v_0, v_1).

As an example, suppose the pause time P is distributed uniformly on $(0, P_{max})$. The cumulative distribution function of P is then $H(t) = t/P_{max}$ for $0 < t < P_{max}$, and $E(P) = P_{max}/2$. Computing the integral in Equation (17) yields

$$H_0(p) = [2pP_{max} - p^2]/P_{max}^2 \tag{18}$$

Inverting yields

$$H_0^{-1}(u) = P_{max}(1 - \sqrt{1-u^2}) \tag{19}$$

Therefore to choose P_0, choose U uniformly on $(0,1)$ and let

$$P_0 = P_{max}(1 - \sqrt{1-U^2}) \tag{20}$$

4 Simulation

4.1 Results (without Pausing)

We simulated a single node traveling in a 1000 m $\times$ 1000 m region. Our first simulation was of the traditional random waypoint mobility model, with a pause time of zero. (In the figures, we refer to this model as *Traditional without Pausing*.) All destinations were chosen uniformly on the region, and the speed for each excursion (in meters per second) was chosen uniformly on the interval (0.01, 20). We note that the initial location and speed were sampled uniformly, rather than from their stationary distributions, in this first simulation. The node traveled for 1000 seconds. The location of the node was updated once per second. This was repeated 10,000 times.

Figure 1 presents histograms for the x-coordinate of the node after one second of travel, during the first 100 seconds, and during the last 100 seconds. Superimposed on the histograms is the stationary density (Equation 5), which has been computed numerically. After one second, it is

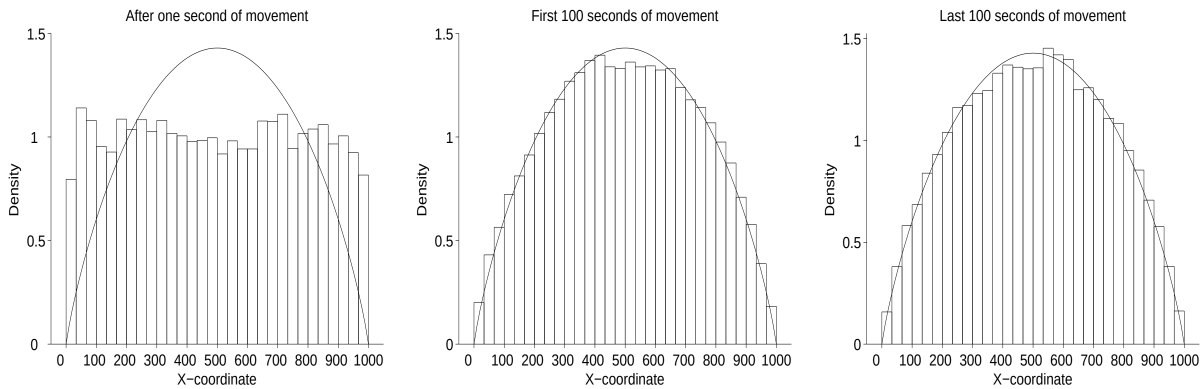

Figure 1: **Traditional without Pausing:** *Left:* x-coordinate after one second of movement. The distribution is nearly uniform, reflecting the distribution of the initial value. *Center*: x-coordinate during first 100 seconds of movement. The stationary distribution has been reached for practical purposes. *Right:* x-coordinate during last 100 seconds of movement. Again, the stationary distribution has been reached for all practical purposes.

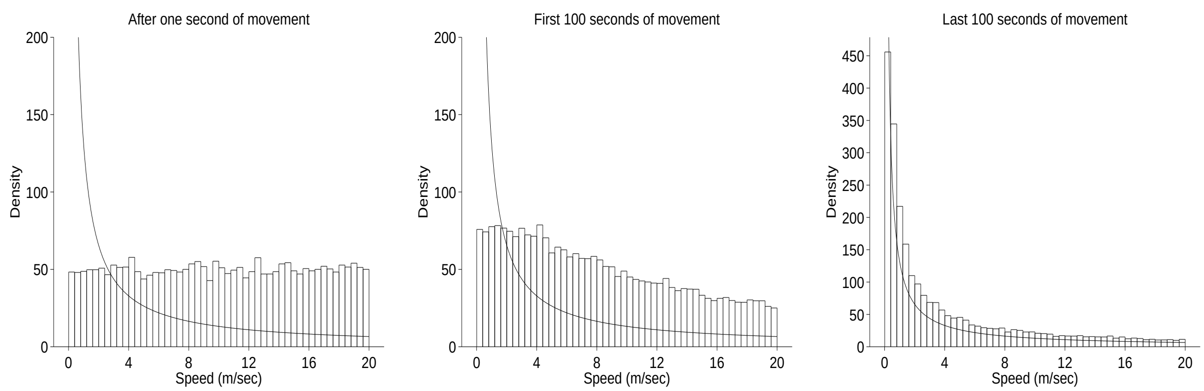

Figure 2: **Traditional without Pausing:** *Left:* Speed after one second of movement. The distribution is nearly uniform, reflecting the distribution of the initial value. *Center*: Speed during first 100 seconds of movement.The distribution has begun to move toward the stationary distribution. *Right:* Speed during the last 100 seconds. The distribution is closer to its stationary value, but still has not converged.

clear that the distribution of the node's *x*-coordinate is close to the uniform distribution of the initial *x*-coordinate. The distribution of the *x*-coordinate converges to the stationary distribution fairly quickly. Over the first 100 seconds of travel, the distribution of the *x*-coordinate is close to its stationary distribution. The distribution over the last 100 seconds is also close to the stationary distribution. Results for the *y*-coordinate (not shown) are similar.

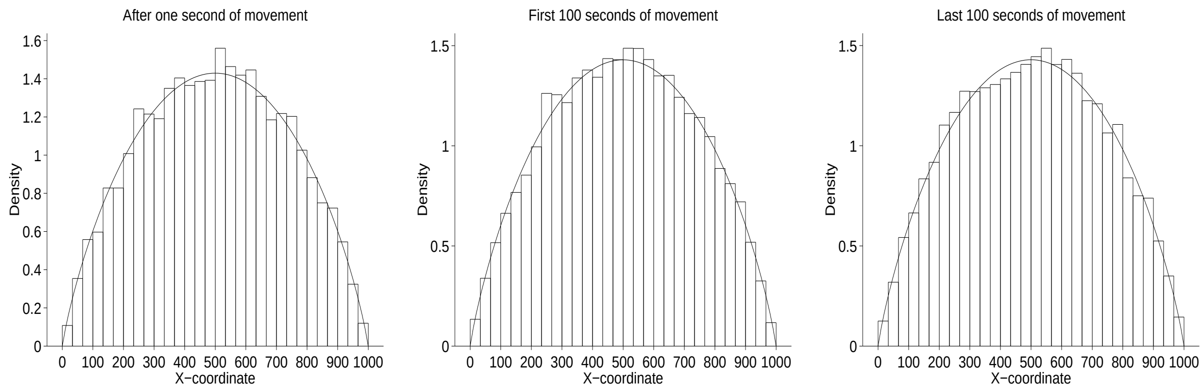

Figure 3: **Stationary without Pausing** *Left:* x-coordinate after one second of movement. *Center:* x-coordinate during first 100 seconds of movement. *Right:* x-coordinate during last 100 seconds of movement. The stationary distribution holds at all times.

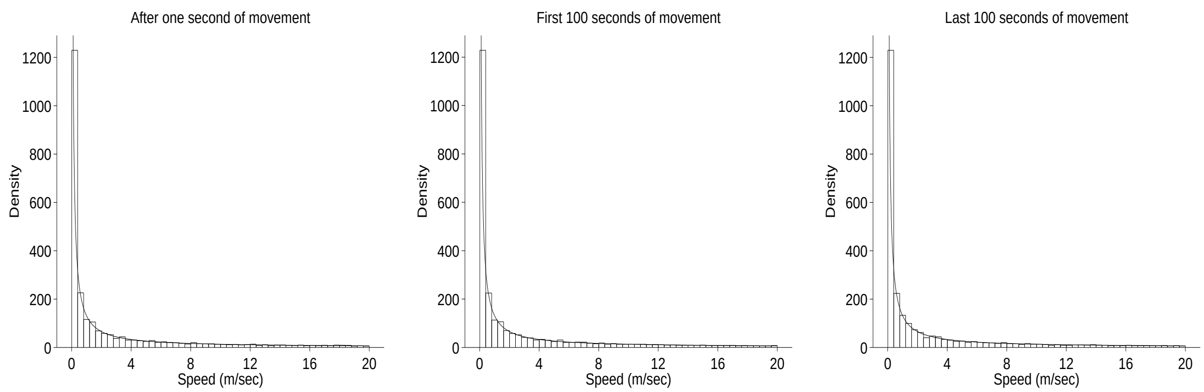

Figure 4: **Stationary without Pausing** *Left:* Speed after one second of movement. *Center*: Speed during first 100 seconds of movement. *Right:* Speed during the last 100 seconds. The stationary distribution holds at all times.

Figure 2 presents histograms for the speed of the node after one second of travel, during the first 100 seconds, and during the last 100 seconds. Superimposed on the histograms is the stationary density (Equation 1). After one second, it is clear that the distribution of the node's speed is close to the uniform distribution of the initial speed. The convergence to the stationary distribution is very slow. Even after 1000 seconds, the distribution is noticeably different from the stationary distribution.

The second simulation was performed in the same way as the first, except that we chose the initial speed and location from their stationary distributions. (In the figures, we refer to this model as *Stationary without Pausing*.) All subsequent speeds and destinations were chosen uniformly. The pause time was zero. Figures 3 and 4 present histograms of the x-coordinate and the speed after one second of travel, during the first 100 seconds, and during the last 100 seconds. The histograms indicate that the speed and location maintain their stationary distributions throughout the simulation.

4.2 Results (with Pausing)

We also simulated a single node traveling via the random waypoint mobility model with a pause time chosen uniformly between 0 and 20 seconds. All other implementation details match the ones discussed in Section 4.1. In our first simulation (*Traditional with Pausing*), we employed the traditional random waypoint mobility model, using an implementation in which nodes began in a moving state and the initial speed and location were sampled uniformly (*mobgen*). In our second simulation (*Stationary with Pausing*), the initial speed, location, and whether the node began in a paused or moving state was chosen from their stationary distributions.

Figure 5 presents histograms for the x-coordinate of the node after one second of travel, during the first 100 seconds, and during the last 100 seconds. Superimposed on the histograms is the stationary density (Equation 16), which has been computed numerically. The results are similar to those in which there was no pausing. After one second, it is clear that the distribution of the node's x-coordinate is close to the uniform distribution of the initial x-coordinate. The distribution of the x-coordinate converges to the stationary distribution fairly quickly. Over the first 100 seconds of travel, the distribution of the x-coordinate is close to its stationary distribution. The distribution over the last 100 seconds is also close to the stationary distribution. Results for the y-coordinate (not shown) are similar.

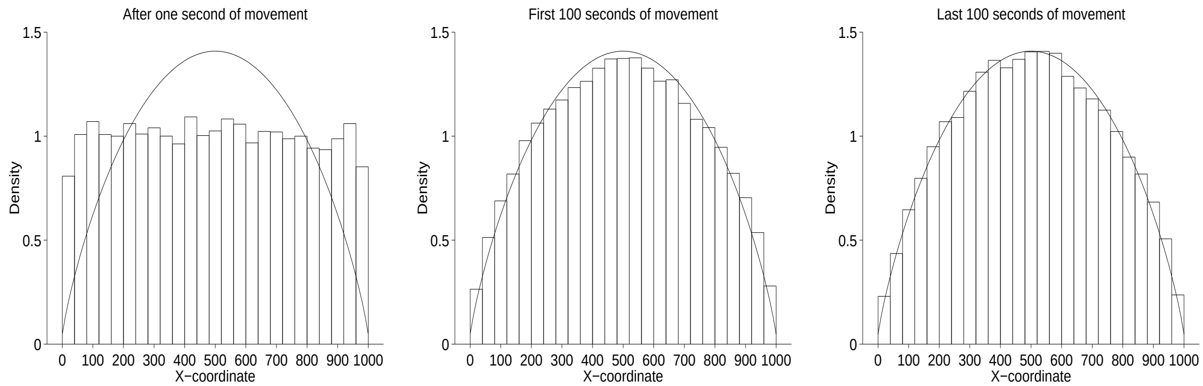

Figure 5: **Traditional with Pausing** *Left:* x-coordinate after one second of movement. The distribution is nearly uniform, reflecting the distribution of the initial value. *Center*: x-coordinate during first 100 seconds of movement. The stationary distribution has been reached for practical purposes. *Right:* x-coordinate during last 100 seconds of movement. Again, the stationary distribution has been reached for all practical purposes.

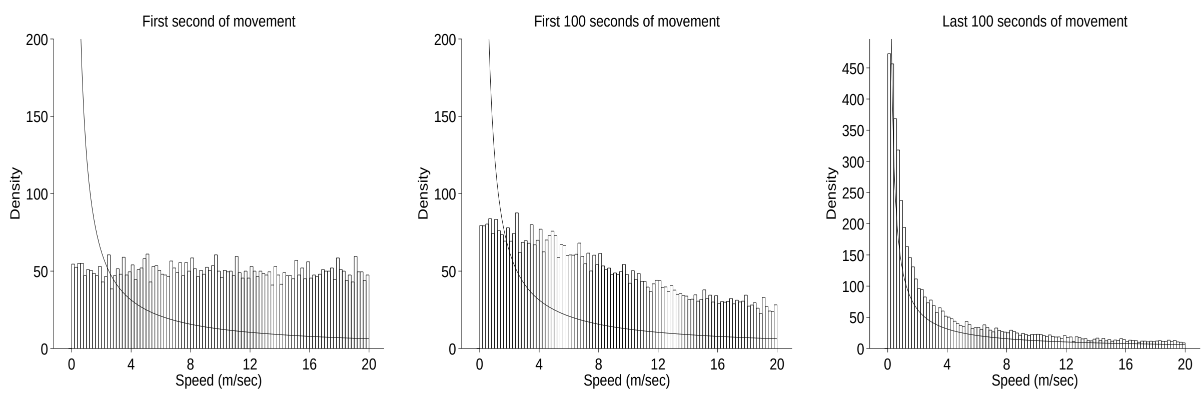

Figure 6: **Traditional with Pausing** *Left:* Speed after one second of movement. The distribution is nearly uniform, reflecting the distribution of the initial value. *Center*: Speed during first 100 seconds of movement.The distribution has begun to move toward the stationary distribution. *Right:* Speed during the last 100 seconds. The distribution is closer to its stationary value, but still has not converged.

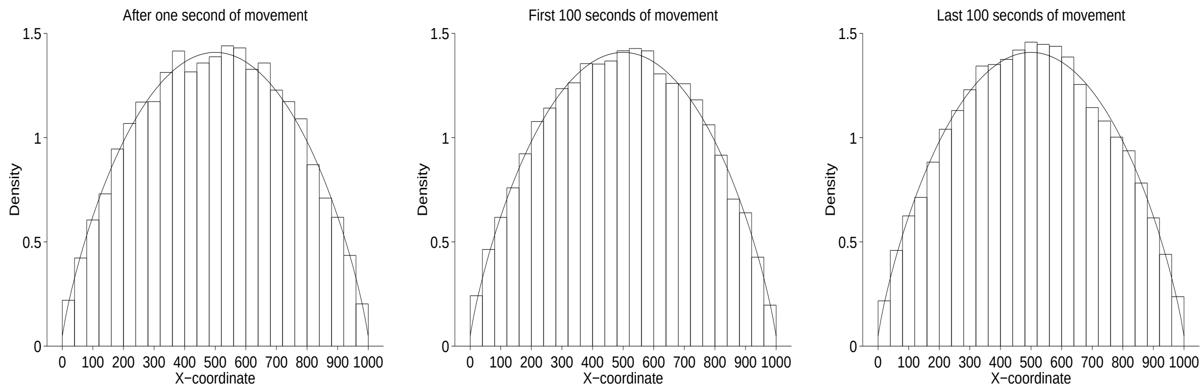

Figure 7: **Stationary with Pausing** *Left:* x-coordinate after one second of movement. *Center:* x-coordinate during first 100 seconds of movement. *Right:* x-coordinate during last 100 seconds of movement. The stationary distribution holds at all times.

When the pause time is non-zero, the stationary density of speed (Equation 15) is a mixture of the continuous density (Equation 1) with an atom of probability, equal to P_{pause}, at zero. We study the distribution of speeds during the times that the node is in motion with a histogram. Then we compute the proportion of time spent paused separately.

Figure 6 presents histograms for the speed of the node after one second of travel, during the first 100 seconds, and during the last 100 seconds, for times when the node was moving. Superimposed on the histograms is the stationary density (Equation 1). After one second, it is clear that the distribution of the node's speed is close to the uniform distribution of the initial speed. Note that we chose to begin the simulation with all nodes in motion (*mobgen*). If we had begun with all nodes paused (*setdest*), all nodes would have had a speed of zero after one second. The convergence to the stationary distribution is very slow. Even after 1000 seconds, the distribution is noticeably different from the stationary distribution.

The second simulation was performed in the same way as the first, except that we chose the initial speed and location from their stationary distributions. All subsequent speeds and destinations were chosen uniformly. The pause time was chosen uniformly on the interval (0, 20). Figures 7 and 8

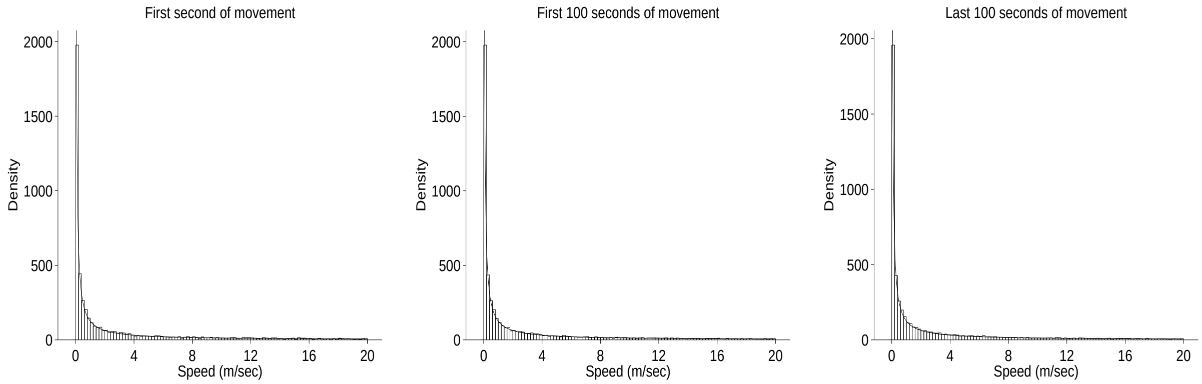

Figure 8: **Stationary with Pausing** *Left:* Speed after one second of movement. *Center*: Speed during first 100 seconds of movement. *Right:* Speed during the last 100 seconds. The stationary distribution holds at all times.

Table 1: Proportion of Time Spent Paused

	First Second	First 100 Seconds	Last 100 Seconds
Traditional	0.0000	0.1063	0.0763
Stationary	0.0498	0.0494	0.0507

present histograms of the *x*-coordinate and the speed after one second of travel, during the first 100 seconds, and during the last 100 seconds. The histograms for speed are restricted to the times that the node was in motion. The histograms indicate that the speed and location maintain their stationary distributions throughout the simulation.

Under the conditions of the simulation, the expected proportion of time spent paused (P_{pause}) is 0.0480. Table 1 presents the proportions of time spent paused in our simulations during the first second, the first 100 seconds, and the last 100 seconds for both the traditional random waypoint mobility model and the model started from the stationary distribution. Under the traditional model, there is no pausing in the first second, which is due to the fact that we chose to start the simulation with the nodes moving (*mobgen*). (If we had chose to start the simulation with the nodes paused (*setdest*), the proportion of time paused in the first second would be 1.0.) Furthermore, in the traditional model, the proportion of time paused is closer to the stationary value of 0.480 during the last 100 seconds than during the first 100 seconds (0.1063 vs. 0.0763), but has still not converged. When the simulation is started from the stationary distribution, however, the proportion of time paused is consistently near the stationary value throughout the simulation.

5 Alternative Distributions for Speed

Figure 4 shows that when speeds are chosen from a uniform distribution with a low minimum speed, then at any given time a large proportion of nodes will be moving very slowly. This was pointed out by Yoon et al. [2] as well. For example, if speed is chosen uniformly on (0.01, 20), and the pause time is zero, it is easy to compute from the stationary density (Equation 1) that on average, half the nodes will be moving at speeds less than 0.45, and 25% of the nodes will be moving at speeds less than 0.07. This can create a nearly stable backbone that can make network performance seem unrealistically good. For this reason, it may be desirable to choose node speeds in a way that avoids having large numbers of slow moving nodes.

An easy solution to the problem is to increase the minimum speed. However, it may be desired to simulate a network in which there are a few slow-moving nodes, but not too many. We present a method for choosing speeds from any desired stationary distribution. Since a node traveling at speed s spends time $1/s$ on a path of length 1, it follows that if speeds are chosen according to a probability density $p(s)$, the stationary density will be proportional to $p(s)/s$. Therefore, any stationary density can be achieved through an appropriate choice of $p(s)$. For example, if

$$p(s) = \frac{2s}{v_1^2 - v_0^2} \text{ for } v_0 < s < v_1 \tag{21}$$

then the stationary density of speed will be uniform on $(v_0,\ v_1)$, since $p(s)/s$ is constant.

To choose speeds so that the distribution will be uniform on $(v_0,\ v_1)$ throughout the simulation, choose the initial speed from the uniform distribution on $(v_0,\ v_1)$ (since that is the stationary distribution), then choose all subsequent speeds from the density given in Equation (21). Note that it is permissible to set $v_0 = 0$, so that arbitrarily slow speeds can be attained.

6 Conclusions

We derive the stationary distributions for the speed, location, and pause time of a node moving in a rectangular area under the random waypoint mobility model. To begin a simulation in the "steady-state" distribution of the random waypoint mobility model, each node must initially determine whether it will begin in a paused or moving state. If the node begins in a paused state, the length of the pause time should be determined via Equation 20. If the node begins in a moving state, the initial location and speed of the node should be determined by using the seven steps given in Section 2.2.

To our knowledge, all published simulation results that use the random waypoint mobility model to compare mobile ad hoc network routing protocols began the simulations either with all nodes paused or with all nodes moving. Only a few published simulation results discard an initial sequence of observations in the hope that the remaining values will closely represent the "steady-state" distribution of the random waypoint mobility model. Our proposed method will guarantee that a simulation begins in the "steady-state" distribution of the random waypoint mobility model.

We present simulation results to verify the correctness of our method. The implementations of our methods (with and without pausing) is available at *http://toilers.mines.edu* for the NS-2 [14] simulator.

References

[1] Camp, T., Boleng, J. and Davies, V. (2001). A survey of mobility models for ad hoc network research. *Wireless Communications & Mobile Computing (WCMC): Special Issue on Mobile Ad Hoc Networking: Research, Trends, and Applications*, vol. 2, no. 5, 483–502.

[2] Yoon, J., Liu, M., and Noble, B. (2003). Random waypoint considered harmful. In *Proceedings of the 21st Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM 2003)*. To appear.

[3] Camp, T., Boleng, J., Williams, B., Wilcox, L., and Navidi, W. (2002). Performance comparison of two location based routing protocols for ad hoc networks. In *Proceedings of the 20th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM 2002)*, 1678-1687.

[4] Johnson, D., and Maltz, D. (1996). Dynamic source routing in ad hoc wireless networks. In *Mobile Computing, T. Imelinsky and H. Korth, eds*, Kluwer Academic Publishers, 153–181.

[5] Broch, J., Maltz D., Johnson, D., Hu, Y., and Jetcheva, J. (1998). Multi-Hop wireless ad hoc network routing protocols, *Proceedings of the ACM/IEEE International Conference on Mobile Computing and Networking (MOBICOM '98)*, 85–97.

[6] Monarch Project, Wireless and Mobility Extensions to ns-2, URL: http://www.monarch.cs.rice.edu/cmu-ns.html. Page accessed on March 26th, 2003.

[7] Royer E. M., Melliar-Smith P. M., and Moser L. E. (2001). An Analysis of the Optimum Node Density for Ad hoc Mobile Networks, *Proceedings of the IEEE International Conference on Communications (ICC 2001)*, 857–861.

[8] Bettstetter, C. and Wagner, C. (2002). The spatial node distribution of the random waypoint mobility model. In *Proceedings of the First German Workshop on Mobile Ad-Hoc Networks (WMAN), GI Lecture Notes in Informatics, P-11*, 41–58.

[9] Bettstetter, C., Resta, G., and Santi, P. (2002). The node distribution of the random waypoint mobility model for wireless ad hoc networks. Technical Report TUM-LKN 2002/01, Technische Universität München, Institute of Communication Network, Munich, Germany, September 2002.

[10] Feller, W. (1970). *An Introduction to Probability and its Applications, Vol II, 2nd ed.* John Wiley & Sons, New York.

[11] Bettstetter, C. (2001). Smooth is better than sharp: A random mobility model for simulation of wireless networks. In *Proceedings of the Fourth ACM International Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM'01)*, Rome, Italy, July 2001.

[12] Bettstetter, C., Hartenstein, H., and Perez-Costa, X. (2002). Stochastic properties of the random waypoint mobility model: Epoch length, direction distribution, and cell change rate. In *Proceedings of the Fifth ACM International Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM'02)*, Atlanta, Georgia, September 28, 2002.

[13] Blough, D., Resta, G., and Santi, P. (2002). A statistical analysis of the long-run node spatial distribution in mobile ad hoc networks. In *Proceedings of the Fifth ACM International Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM'02)*, Atlanta, Georgia, September 28, 2002.

[14] The VINT Project, The Network Simulator - ns-2, URL: http://www.isi.edu/nsnam/ns/. Page accessed on April 12th, 2003.