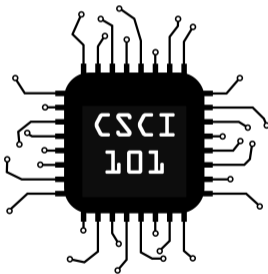


# Recursion

Calling a Function from Within Itself



# Recursion: What is it?

Recursive functions are functions which rely on themselves to calculate part of the answer. Recursive functions usually have a base case that causes the recursion to end. Here is an example as a story:

A child couldn't sleep, so her mother told a story about a little frog,  
who couldn't sleep, so the frog's mother told a story about a little bear,  
who couldn't sleep, so the bear's mother told a story about a little weasel  
...who fell asleep.  
...and the little bear fell asleep;  
...and the little frog fell asleep;  
...and the child fell asleep.

# Recursion: What is it?

## Base Case

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who couldn't sleep, so the frog's mother told a story about a little bear,  
who couldn't sleep, so the bear's mother told a story about a little weasel  
...who fell asleep.  
...and the little bear fell asleep;  
...and the little frog fell asleep;  
...and the child fell asleep.

# Recursion: What is it?

## Recursive Part

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who couldn't sleep, so the frog's mother told a story about a little bear,  
who couldn't sleep, so the bear's mother told a story about a little weasel  
...who fell asleep.  
...and the little bear fell asleep;  
...and the little frog fell asleep;  
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# Recursive Functions in Python

Consider the factorial operation.

$$n! = n \times (n - 1) \times (n - 2) \times \cdots \times 1$$

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- **Base case:**  $0! = 1$
- **Recursive part:**  $n! = n(n - 1)!$

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- **Recursive part:**  $n! = n(n - 1)!$

To code this as a recursive function in Python, we could do:

```
def fact(n):  
    if n == 0:           # base case  
        return 1  
    return n*fact(n-1)  # recursive part
```

# Recursion in Practicality: Euclid's GCD

The GCD of  $a$  and  $b$  is:

- $a$  if  $b = 0$
- $\text{gcd}(b, a \bmod b)$  otherwise

More info about why this is so can be found at

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Implementation in Python:

```
def gcd(a, b):  
    if b == 0:                # base case  
        return a  
    return gcd(b, a % b)     # recursive part
```

## Practice: Fibonacci Numbers

The  $n$ -th Fibonacci number,  $F(n)$ , is:

- $n$  if  $n = 0$  or  $n = 1$
- $F(n - 1) + F(n - 2)$  otherwise



**Try it yourself:** Implement a Python function which calculates the  $n$ -th Fibonacci number recursively.